



A New Classes Of Strongly ξ^*_3 -Continuous Maps In Generalized Binary Ideal Topological Spaces

Pooja Mahadev Khatkale¹, Dr. Sonu Kumar², Dinesh Kute³ and Dr. Nazir Ahmad Ahengar⁴

^{1,2}Department of Mathematics, Asian International University, Manipur – 795140, India

³Department of Applied Sciences and Humanities, Pimpri Chinchwad College of Engineering, Pune -44, India

⁴Department of Mathematics, School of Engineering & Technology, Pimpri Chinchwad University, Pune-412106, India

¹Email:ID: poojakhatkale9@gmail.com/

Email:ID: dineshkute90@gmail.com/

Email:ID: nzrhmd97@gmail.com

Cite This Paper as: Pooja Mahadev Khatkale, Dr. Sonu Kumar, Dinesh Kute and Dr. Nazir Ahmad Ahengar (2026) A New Classes Of Strongly ξ^*_3 -Continuous Maps In Generalized Binary Ideal Topological Spaces The Journal of African Development 1, Vol.7, No.1, 1318-1321

KEYWORDS

Strongly ξ -continuous, strongly ξ_3 -continuous maps, ξ^*_3 -continuous, strongly ξ^*_3 -continuous

Mathematics Subject Classification: 54C05-54C08.

ABSTRACT

In this paper, we introduced the several generalized forms of continuous maps such as ξ^*_3 -continuous and strongly ξ^*_3 -continuous maps in generalized binary ideal topological spaces and investigate various relationships of these maps by demonstrating the use of some counter examples....

1. INTRODUCTION

In recent study, topology in data mining plays a significant role Pawlak, (1991), Kırbas and Aslım (2009). Information systems are fundamental instruments for generating information understanding in any real-life sector, and topological information collection structures are appropriate mathematical models for both quantitative as well as the qualitative information mathematics. This has an impact in digital topology and computer science Khalimsky, Kopperman and Meyer (1990), Kong, Kopperman and Meyer (1991), Kovalsky and Kopperman (1994) and Moore and Peters (2005), quantum, high-energy, particle physics and superstring theory Landi, (1997) and Svozil, (1987).

Initially, Kuratowski (1930) introduced the concept of ideal topological space. The concepts of semi-open and semi-continuous maps is introduced and studied by Levine (1963). Further, the same author Levin (1970) studied generalized closed sets in topology and investigates various properties. The notion of generalized topology was introduced by Csaszar (2001). Hatir and Noiri (2006) and verified several properties of β -I-open sets and studied the several results of almost-I-continuities. Jafaril and Rajesh (2011) studied the concept of g-closed sets with respect of ideals and studied various characterizations. Rodyna and Deena (2013) studied several generalized forms of open sets with respect of ideals and categories them suitably. An ideal in generalized topological space is further defined by Maitra and Tripathi (2014) and obtained important properties of local function in generalized ideal topological spaces.

Egenhofer (1991) discussed the very useful concept for binary topological relations. Gevorgyan (2016) studied the group of continuous binary operations on a topological space and established its relationship with the group of homeomorphisms. Chen et al. (2018) demonstrated the dynamics on binary relations over topological spaces. Nithyanantha and Thangavelu (2011) studied binary topology and investigate various characterizations.

In this paper we developed the concept of ξ^*_3 -continuous and strongly ξ^*_3 -continuous maps. The significance of results has been shown by several counter examples. Some require basic definitions, concepts of ξ -topological, $I\xi_T$ and notations

arediscussed in Section 2. The concept of strongly $\xi_{\mathfrak{S}}^*$ -continuous maps is discussed in Section 3. The conclusion is given in Section 4.

2. Preliminaries

In this portion, we discussed few require and important definitions, concepts of ξ -topological, $I\xi_T S$ and some notations.

Definition 2.1: Suppose Y_1 and Y_2 are any two non-void sets. Then ξ_T from Y_1 to Y_2 is a binary structure $\xi \subseteq \wp(Y_1) \times \wp(Y_2)$ satisfying the conditions i.e., $(\emptyset, \emptyset), (Y_1, Y_2) \in \xi$ and If $\{(L_\alpha, M_\alpha); \alpha \in \Gamma\}$ is a family of elements of ξ , then $(\bigcup_{\alpha \in \Gamma} L_\alpha, \bigcup_{\alpha \in \Gamma} M_\alpha) \in \xi$. If ξ is ξ_T from Y_1 to Y_2 , then (Y_1, Y_2, ξ) is known as ξ -topological space ($\xi_T S$) and the elements of ξ are known as the ξ -open subsets of (Y_1, Y_2, ξ) . The elements of $Y_1 \times Y_2$ are known as ξ -points.

Definition 2.2: Let Y_1 and Y_2 be any two non-void set and $(L_1, M_1), (L_2, M_2)$ are the elements of $\wp(Y_1) \times \wp(Y_2)$. Then $(L_1, M_1) \subseteq (L_2, M_2)$ only if $L_1 \subseteq L_2$ and $M_1 \subseteq M_2$.

Remark 2.1: Let $\{T_\alpha; \alpha \in \Lambda\}$ be the family of ξ_T from Y_1 to Y_2 . Then, $\bigcap_{\alpha \in \Lambda} T_\alpha$ is also ξ_T from Y_1 to Y_2 . Further $\bigcup_{\alpha \in \Lambda} T_\alpha$ need not be a ξ_T .

Definition 2.3: Let (Y_1, Y_2, ξ) be a $\xi_T S$ and $L \subseteq Y_1, M \subseteq Y_2$. Then (L, M) is called ξ -closed in (Y_1, Y_2, ξ) if $(Y_1 \setminus L, Y_2 \setminus M) \in \xi$.

Proposition 2.1: Let (Y_1, Y_2, ξ) is $\xi_T S$. Then (Y_1, Y_2) and (\emptyset, \emptyset) are ξ -closed sets. Similarly if $\{(L_\alpha, M_\alpha); \alpha \in \Gamma\}$ is a family of ξ -closed sets, then $(\bigcap_{\alpha \in \Gamma} L_\alpha, \bigcap_{\alpha \in \Gamma} M_\alpha)$ is ξ -closed.

Proposition 2.2: Let $(L, M) \subseteq (Y_1, Y_2)$. Then (L, M) is ξ -open in (Y_1, Y_2, ξ) iff $(L, M) = I_\xi(L, M)$ and (L, M) is ξ -closed in (Y_1, Y_2, ξ) iff $(L, M) = Cl_\xi(L, M)$.

Definition 2.4: Any non-empty collection \mathfrak{S} of subsets of $Y_1 \times Y_2$ is an ideal only if it satisfies the two important axioms, i.e. if $(L, M) \in \mathfrak{S}$ and $(P, Q) \subseteq (L, M)$ then $(P, Q) \in \mathfrak{S}$ and If $(L, M) \in \mathfrak{S}$ and $(P, Q) \in \mathfrak{S}$ then $(L \cup P, M \cup Q) \in \mathfrak{S}$. Let ξ be ξ_T and \mathfrak{S} be an ideal, then $(Y_1, Y_2, \xi, \mathfrak{S})$ is said to be an ideal ξ -topological space ($I\xi_T S$).

Example 2.2: Let (Y_1, Y_2, ξ) be $\xi_T S$. The collection $\mathfrak{S} = \emptyset$ and $\mathfrak{S} = \wp(Y_1) \times \wp(Y_2)$ are also ideals on $Y_1 \times Y_2$.

Definition 2.5: Let $(Y_1, Y_2, \xi, \mathfrak{S})$ be $I\xi_T S$ and $(L, M) \subseteq Y_1 \times Y_2$. Then the set $(L, M)^*(\mathfrak{S}) = \{(x, y) \in Y_1 \times Y_2 / (U \cap L, V \cap M) \notin \mathfrak{S} \text{ for every nbd } (U, V) \text{ of } (x, y)\}$ is known as the local function of (L, M) in the respect of \mathfrak{S} and ξ . We normally denote $(L, M)^*$ instead of $(L, M)^*(\mathfrak{S})$ to avoid any confusion.

Definition 2.6: Let $(Y_1, Y_2, \xi, \mathfrak{S})$ be $I\xi_T S$ and $(L, M) \subseteq Y_1 \times Y_2$. Then (L, M) is known as $\xi_{\mathfrak{S}}$ -semi-open if for any ξ -open set $(U, V), (L, M) \setminus Cl_\xi(U, V) \in \mathfrak{S}$ whenever, $(U, V) \setminus (L, M) \in \mathfrak{S}$. Likewise (L, M) is known as $\xi_{\mathfrak{S}}-\alpha$ -openif for any ξ -open set $(U, V), (L, M) \setminus I_\xi(Cl_\xi(U, V)) \in \mathfrak{S}$ whenever, $(U, V) \setminus (L, M) \in \mathfrak{S}$.

Definition 2.7: Let $(Y_1, Y_2, \xi, \mathfrak{S})$ be $I\xi_T S$ and $(L, M) \subseteq Y_1 \times Y_2$. Then (L, M) is known as $\xi_{\mathfrak{S}}$ -pre-open if for any ξ -open set $(U, V), (U, V) \setminus Cl_\xi(L, M) \in \mathfrak{S}$ whenever, $(L, M) \setminus (U, V) \in \mathfrak{S}$. Likewise (L, M) is known as $\xi_{\mathfrak{S}}-\beta$ -openif for any ξ -open set (U, V) such that $(U, V) \setminus I_\xi(Cl_\xi(L, M)) \in \mathfrak{S}$ whenever, $(L, M) \setminus (U, V) \in \mathfrak{S}$.

Definition 2.8: Let $(Y_1, Y_2, \xi, \mathfrak{S})$ be $I\xi_T S$ and $(L, M) \subseteq Y_1 \times Y_2$. Then (L, M) is called $\xi_{\mathfrak{S}}^*$ -open set if $(U, V) \setminus (L, M) \in \mathfrak{S}$ whenever, $(L, M) \subseteq (U, V)$, where (U, V) is ξ -open set.

Definition 2.9: Let $(Y_1, Y_2, \xi, \mathfrak{S})$ be $I\xi_T S$ and $(L, M) \subseteq Y_1 \times Y_2$. Then (L, M) is called $\xi_{\mathfrak{S}}^*$ -semi-open set if $(U, V) \setminus I_\xi(L, M) \in \mathfrak{S}$ whenever, $(L, M) \subseteq (U, V)$, where (U, V) is ξ -open set. Similarly (L, M) is called $\xi_{\mathfrak{S}}^*-\alpha$ -open set if $(U, V) \setminus Cl_\xi(I_\xi(L, M)) \in \mathfrak{S}$ whenever, $(L, M) \subseteq (U, V)$, where (U, V) is ξ -open set.

Definition 2.10: Let $(Y_1, Y_2, \xi, \mathfrak{S})$ be $I\xi_T S$ and $(L, M) \subseteq Y_1 \times Y_2$. Then (L, M) is called $\xi_{\mathfrak{S}}^*$ -pre-open set if $(U, V) \setminus Cl_\xi(L, M) \in \mathfrak{S}$ whenever, $(L, M) \subseteq (U, V)$, where (U, V) is ξ -open set. Similarly (L, M) is called $\xi_{\mathfrak{S}}^*-\beta$ -open set if $(U, V) \setminus I_\xi(Cl_\xi(L, M)) \in \mathfrak{S}$ whenever, $(L, M) \subseteq (U, V)$, where (U, V) is ξ -open set.

Definition 2.11: If (Z, \mathcal{T}) be G_T and $(Y_1, Y_2, \xi, \mathfrak{S})$ is $I\xi_T S$. Then the mapping $\mathcal{F}: (Z, \mathcal{T}) \rightarrow Y_1 \times Y_2$ is known as $\xi_{\mathfrak{S}}-\text{semi } (\xi_{\mathfrak{S}}-\alpha)$ continuous map if $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}_{\mathfrak{S}}-\text{semi } (\mathcal{T}_{\mathfrak{S}}-\alpha)$ open in $(Z, \mathcal{T}) \quad \forall \xi$ -open sets $(L, M) \in (Y_1, Y_2, \xi)$.

Definition 2.12: If (Z, \mathcal{T}) be G_T and $(Y_1, Y_2, \xi, \mathfrak{S})$ is $I\xi_T S$. Then the mapping $\mathcal{F}: (Z, \mathcal{T}) \rightarrow Y_1 \times Y_2$ is known as strongly $\xi_{\mathfrak{S}}-\text{semi } (\text{strongly } \xi_{\mathfrak{S}}-\alpha)$ continuous map if $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}_{\mathfrak{S}}-\text{semi } (\mathcal{T}_{\mathfrak{S}}-\alpha)$ clopen in $(Z, \mathcal{T}) \quad \forall \xi$ -set $(L, M) \in (Y_1, Y_2, \xi)$.

Definition 2.13: If (Z, \mathcal{T}) be $G_T S$ and $(Y_1, Y_2, \xi, \mathfrak{S})$ is $I\xi_T S$. Then the mapping $\mathcal{F}: (Z, \mathcal{T}) \rightarrow Y_1 \times Y_2$ is known as $\xi_{\mathfrak{S}}-\text{pre } (\xi_{\mathfrak{S}}-\beta)$ continuous map if $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}_{\mathfrak{S}}-\text{pre } (\mathcal{T}_{\mathfrak{S}}-\beta)$ open in $(Z, \mathcal{T}) \quad \forall \xi$ -open sets $(L, M) \in (Y_1, Y_2, \xi)$.

Definition 2.14: If (Z, \mathcal{T}) be $G_T S$ and $(Y_1, Y_2, \xi, \mathfrak{S})$ is $I\xi_T S$. Then the mapping $\mathcal{F}: (Z, \mathcal{T}) \rightarrow Y_1 \times Y_2$ is known as strongly $\xi_{\mathfrak{S}}-\text{pre } (\text{strongly } \xi_{\mathfrak{S}}-\beta)$ continuous map if $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}_{\mathfrak{S}}-\text{pre } (\mathcal{T}_{\mathfrak{S}}-\beta)$ clopen in $(Z, \mathcal{T}) \quad \forall \xi$ -set $(L, M) \in (Y_1, Y_2, \xi)$.

3. Strongly ξ^*_3 -Continuous Maps

In this section, we established the relationships strongly ξ^*_3 -continuous maps in $I\xi_T S$ and some other maps. The results have been shown by making the use of some counter examples.

Definition 3.1: If (Z, \mathcal{T}) be $G_T S$ and $(Y_1, Y_2, \xi, \mathfrak{S})$ is $I\xi_T S$. Then the mapping $\mathcal{F}: (Z, \mathcal{T}) \rightarrow Y_1 \times Y_2$ is known as ξ^*_3 -continuous map if $\mathcal{F}^{-1}(L, M)$ is \mathcal{T}^*_3 -open in $(Z, \mathcal{T}) \forall \xi$ -open set $(L, M) \in (Y_1, Y_2, \xi)$.

Definition 3.2: If (Z, \mathcal{T}) be G_T and $(Y_1, Y_2, \xi, \mathfrak{S})$ is $I\xi_T S$. Then the mapping $\mathcal{F}: (Z, \mathcal{T}) \rightarrow Y_1 \times Y_2$ is known as **strongly ξ^*_3 -continuous map** if $\mathcal{F}^{-1}(L, M)$ is \mathcal{T}^*_3 -clopen in $(Z, \mathcal{T}) \forall \xi$ -set $(L, M) \in (Y_1, Y_2, \xi)$.

Example 3.1: Let $Z = \{1, 2, 3\}$, $Y_1 = \{a_1, a_2\}$ and $Y_2 = \{b_1, b_2\}$. Then clearly $\mathcal{T} = \{\emptyset, \{1, 2\}, \{2, 3\}, Z\}$ is G_T , $\mathfrak{S} = \{\emptyset, \{2\}, \{3\}, \{2, 3\}\}$ is an ideal on Z and $\xi = \{(\emptyset, \emptyset), (\{a_1\}, \{b_1\}), (\{a_2\}, \{b_2\}), (Y_1, Y_2)\}$ is ξ_T from Y_1 to Y_2 . Consider $\mathcal{F}: (Z, \mathcal{T}) \rightarrow Y_1 \times Y_2$ defined as $\mathcal{F}(1) = (a_1, b_1) = \mathcal{F}(3)$ and $\mathcal{F}(2) = (a_2, b_2)$. Therefore $\mathcal{F}^{-1}(\emptyset, \emptyset) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{a_1\}, \{b_1\}) = \{1, 3\}$, $\mathcal{F}^{-1}(\{a_1\}, \{b_2\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{a_2\}, \{b_2\}) = \{2\}$, $\mathcal{F}^{-1}(\{\emptyset\}, \{b_1\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{\emptyset\}, \{b_2\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{\emptyset\}, \{Y_2\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{a_1\}, \{\emptyset\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{a_1\}, \{b_2\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{a_2\}, \emptyset) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{a_2\}, \{b_1\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{a_2\}, \{b_2\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{Y_1\}, \{\emptyset\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{Y_1\}, \{b_1\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{Y_1\}, \{b_2\}) = \{\emptyset\}$ and $\mathcal{F}^{-1}(Y_1, Y_2) = \{Z\}$. Hence we see that the inverse image of every ξ -set is \mathcal{T}^*_3 -clopen set in (Z, \mathcal{T}) . Thus \mathcal{F} is strongly ξ^*_3 -continuous map.

Remark 3.1: ξ (ξ -semi, ξ - α , ξ -pre, ξ - β) continuous map $\Rightarrow \neq$ strongly ξ^*_3 -continuous map

Proof: Quite easy while the converse can be illustrated as follows.

Example 3.2: The map, \mathcal{F} in Example 4.1 is strongly ξ^*_3 -continuous map but not ξ (ξ -semi, ξ - α , ξ -pre, ξ - β) continuous map.

Proposition 3.1: Strongly ξ^*_3 -continuous map $\Rightarrow \neq$ ξ_3 -semi (ξ_3 - α) continuous map

Proof: Suppose (L, M) be ξ -set and \mathcal{F} be strongly ξ^*_3 -continuous map. Therefore $\mathcal{F}^{-1}(L, M)$ is \mathcal{T}^*_3 -clopen set in (Z, \mathcal{T}) . Since every \mathcal{T}^*_3 -clopen set is \mathcal{T}_3 -semi (\mathcal{T}_3 - α) open, therefore $\mathcal{F}^{-1}(L, M)$ is \mathcal{T}_3 -semi (\mathcal{T}_3 - α) open set in (Z, \mathcal{T}) . Hence \mathcal{F} is ξ_3 -semi (ξ_3 - α) continuous map.

The converse can be illustrated as follows.

Example 3.3: Let $Z = \{1, 2, 3, 4\}$, $Y_1 = \{a_1, a_2\}$ and $Y_2 = \{b_1, b_2\}$. Then clearly $\mathcal{T} = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, Z\}$ is G_T , $\mathfrak{S} = \{\emptyset, \{2, 3\}\}$ is an ideal on Z and $\xi = \{(\emptyset, \emptyset), (\{a_1\}, \{b_1\}), (\{a_2\}, \{b_2\}), (\{a_2\}, \{Y_2\}), (Y_1, Y_2)\}$ is ξ_T from Y_1 to Y_2 . Consider $\mathcal{F}: (Z, \mathcal{T}) \rightarrow Y_1 \times Y_2$ defined as $\mathcal{F}(3) = (a_1, b_1) = \mathcal{F}(4)$. Hence we see that the inverse image of every ξ -set is \mathcal{T}_3 -semi (\mathcal{T}_3 - α) open set but not \mathcal{T}^*_3 -clopen set in (Z, \mathcal{T}) . Thus \mathcal{F} is ξ_3 -semi (ξ_3 - α) continuous map but not strongly ξ^*_3 -continuous map.

Proposition 3.2: Strongly ξ^*_3 -continuous map $\Rightarrow \neq$ ξ_3 -pre (ξ_3 - β) continuous map.

Proof: Suppose (L, M) be ξ -set and \mathcal{F} is strongly ξ^*_3 -continuous map. Therefore $\mathcal{F}^{-1}(L, M)$ is \mathcal{T}^*_3 -clopen set in (Z, \mathcal{T}) . Since every \mathcal{T}^*_3 -clopen set is \mathcal{T}_3 -pre (\mathcal{T}_3 - β) open, therefore $\mathcal{F}^{-1}(L, M)$ is \mathcal{T}_3 -pre (\mathcal{T}_3 - β) open set in (Z, \mathcal{T}) . Hence \mathcal{F} is ξ_3 -pre (ξ_3 - β) continuous map.

The converse is illustrated as follows.

Example 3.4: Let $Z = \{1, 2, 3, 4\}$, $Y_1 = \{a_1, a_2\}$ and $Y_2 = \{b_1, b_2\}$. Then clearly $\mathcal{T} = \{\emptyset, \{1\}, \{3\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{1, 2, 3\}, \{1, 3, 4\}, \{1, 2, 4\}, \{2, 3, 4\}, Z\}$ is G_T , $\mathfrak{S} = \{\emptyset, \{1\}\}$ is an ideal on Z and $\xi = \{(\emptyset, \emptyset), (\{a_1\}, \{b_1\}), (\{a_2\}, \{b_2\}), (\{a_2\}, \{Y_2\}), (Y_1, Y_2)\}$ is ξ_T from Y_1 to Y_2 . Consider $\mathcal{F}: (Z, \mathcal{T}) \rightarrow Y_1 \times Y_2$ defined as $\mathcal{F}(3) = (a_1, b_1) = \mathcal{F}(4)$. Hence we see that the inverse image of every ξ -set is \mathcal{T}_3 -pre (\mathcal{T}_3 - β) open set but not \mathcal{T}^*_3 -clopen set in (Z, \mathcal{T}) . Thus \mathcal{F} is ξ_3 -pre (ξ_3 - β) continuous map but not strongly ξ^*_3 -continuous map.

5. CONCLUSION

We introduced and study useful concept of ξ^*_3 -continuous and strongly ξ^*_3 -continuous and established the relationship between above discusses maps and several other maps like ξ , ξ -semi, ξ -pre, ξ - α , ξ - β , ξ_3 -semi, ξ_3 -pre, ξ_3 - α , ξ_3 - β and ξ^*_3 -continuous maps etc. In the present direction, we have categorised maps in generalized binary ideal topological spaces and investigated the behaviour of presented maps by utilizing ideal. The significant of results have been shown by several counter examples

References

1. Chen, C. C., Conejero, J. A, Kostic, M, Murillo-Arcila, M., (2018). Dynamics on Binary Relations over Topological Spaces. Symmetry, 10: 211. <https://doi.org/10.3390/sym10060211>

2. Csaszar, A., (2001). Generalized topology, generalized continuity, *Acta Mathematica Hungarica*, 96, 351-357.
3. Egenhofer, M. J., (1991). Reasoning about binary topological relations. *Symposium on Spatial Databases SSD 1991: Advances in Spatial Databases*, 141-160.
4. Gevorgyan, P. S., (2016). Groups of binary operations and binary G-spaces. *Topology and its Applications*, 201, 18–28.
5. Hatir, E., Noiri, T., (2006). Decompositions of continuity and complete continuity. *Acta Mathematica Hungarica*, 4, 281–287.
6. Jafari1, Rajesh, S. N., (2011). Generalized Closed Sets with Respect to an Ideal. *European Journal of Pure and Applied Mathematics*, 2, 147-151.
7. Kırbas, H., & Aslım, G., (2009). Decompositions of continuity and some weak forms of continuity, *Chaos, Solitons and Fractals*, 41, 1684–1690.
8. Khalimsky, E. D., Kopperman R., & Meyer P. R., (1990). Computer graphics and connected topologies an finite ordered sets. *Topology and its applications*, 36, 1–17.
9. Kong, T. Y., Kopperman R., & Meyer P. R., (1991). A topological approach to digital topology. *American Mathematical Monthly*, 98, 901–17.
10. Kovalsky, V., & Kopperman, R., (1994). Some topology-based imaged processing algorithms, *Annals of the New York Academy of Sciences*, 728, 174–82.
11. Kuratowski, K., (1930). *Topologie I*, Warszawa
12. Landi, G., (1997). *An introduction to non-commutative spaces and their geometrics (lecture notes in physics)*, New York Springer Verlag.
13. Levine N., (1961). A decomposition of continuity in topological spaces. *American Mathematical Monthly*, 68, 44–6.
14. Levine, N., (1963). Semi open sets and semi continuity in topological spaces, *American Mathematical Monthly*, 70, 36-41.
15. Levine, N., (1970). Generalized closed sets in Topology, *Rendiconti del Circolo Matematico di Palermo*, 2, 89-96.
16. Maitra, J. K., & Tripathi, H. K., (2014). Local function in generalized ideal topological spaces, *Vislesana*, 1, 191-195.
17. Michael, F., (2013). On semi-open sets with respect to an ideal, *European Journal of Pure and Applied Mathematics.*, 6, 53 – 58.
18. Moore, E. L. F., & Peters, T. J., (2005). Computational topology for geometric design and molecular design. In: Ferguson DR., Peters TJ., editors. *Mathematics in industry challenges and frontiers SIAM*
19. Njastad, O., (1965). On some classes of nearly open sets, *Pacific Journal of Mathematics*, 15, 961–970.
20. NithyananthaJothi S., & Thangavelu P., (2011). On binary topological spaces, *Pacific-Asian Journal of Mathematics* 2, 133-138.
21. Pawlak, Z., (1991). *Rough sets: theoretical aspects of reasoning about data. System theory, knowledge engineering and problem solving*, vol. 9. Dordrecht: Kluwer
22. Rodyna, A. H., & Deena, A. K., (2013). Types of Generalized Open Sets with Ideal, *International Journal of Computer Applications*, 4, 0975-8887.
23. Rodyna, A. H., (2013). Pre-open sets with ideal, *European Journal of Scientific Research*, 1, 99 -101.
24. Svozil, K., (1987). Quantum field theory on fractal space–time: a new regularization method. *Journal of Physics A Mathematical and General*, 20, 3861–75.