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On Almost Completely ξ-θ-Semi-Continuous Maps in Generalized Binary Topological Spaces

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KEYWORDS

continuous maps, almost completely ξ -θ-semi-continuous maps.

ABSTRACT

In this paper the concepts of several types of almost completely ξ -θ-semi-continuous maps in generalized binary topological spaces are introduced and all the possible relationships of these maps have been discussed and established by making the use of some counter examples....

1. INTRODUCTION

Hatir and Noiri [10] studied d–b-continuous functions and obtained several results related to continuity. Continuity is most important concept in Mathematics and many different generalized forms of continuity have been studied and investigated. Levine [14] introduced weakly continuous functions and established some new results. Further, Son et al. [22] introduced weakly clopen and almost clopen functions. Egenhofer [11] discussed the very useful concept for binary topological relations. Gevorgyan [9] studied thegroup of continuous binary operations on a topological space and established its relationship with the group of homeomorphisms. Chen et al. [5] demonstrated the dynamics on binary relations over topological spaces.

Benchalli S.S and Umadevi I Neeli Nour T.M [3, 19] studied the concept of totally semi-continuous functions and semi-totally continuous functions in topological spaces and verify the certain properties of the concept. The authors Arya,S. P., Gupta,R Anuradha, Baby Chacko and Singh D [1-2,21] introduced the concept of strongly continuous functions and almost perfectly continuous functions in topological spaces and established the various significant results. Nithyanantha and Thangavelu [18] introduced the concept of binary topology between two sets and investigate some of the basic properties, where a binary topology from X to Y is a binary structure satisfying certain axioms that are analogous to the axioms of topology. Jamal M. Mustafa [11] studied binary generalized topological spaces and investigate the various relationships of the maps so discussed with some other maps

Contribution

As outline, the concept of ξ-θ-semi-continuous maps, almost completely ξ-θ-semi-continuous maps, θ-semi-totally ξ-continuous maps strongly ξ-θ-semi-continuous maps, almost perfectly ξ-θ-semi-continuous maps, totally ξ-θ-semi-continuous maps have been introduced and studied different properties of these maps in this paper. The significant of results have been shown by several counter examples.

Organization

The rest of the paper structured as follows: Some require basic definitions, concepts of ξ-topological and notations are discussed in Section 2. The section 3, headed by the concept of *almost completely ξ-θ-semi-open-continuous maps* in which several other maps are studied and established the relationships. Section 4 concludes the paper. Throughout the paper $\wp(\Upsilon)$ denotes the power set of Υ..

2. Preliminaries

Some require and important definitions and concepts of *ξ-topological space* and notations have been given in this portion

Definition 2.1: Let Υ_1 and Υ_2 be any two non-void sets. Then ξ-topology (ξ_T) from Υ_1 to Υ_2 is a binary structure $\xi \subseteq \wp(\Upsilon_1) \times \wp(\Upsilon_2)$ satisfying the conditions i.e. $(\emptyset, \emptyset)$, $(\Upsilon_1, \Upsilon_2) \in \xi$ and If $\{(L_\alpha,\ M_\alpha)\ ;\ \alpha \in \Gamma\}$ is a family of elements of ξ, then $(\bigcup_{\alpha\in\Gamma} L_\alpha, \bigcup_{\alpha\in\Gamma} M_\alpha) \in \xi$. If ξ is ξ_T from Υ_1 to Υ_2, then $(\Upsilon_1, \Upsilon_2, \xi)$ is called a ξ-topological space ($\xi_T S$) and the elements of ξ are called the ξ-open subsets of $(\Upsilon_1, \Upsilon_2, \xi)$. The elements of $\Upsilon_1 \times \Upsilon_2$ are called simply ξ-points

Definition 2.2: Let Υ_1 and Υ_2 be any two non-void set and (L_1, M_1), (L_2, M_2) are the elements of $\wp(\Upsilon_1) \times \wp(\Upsilon_2)$. Then $(L_1, M_1) \subseteq (L_2, M_2)$ only if $L_1 \subseteq L_2$ and $M_1 \subseteq M_2$.

Remark 2.1: Let $\{T_\alpha\ ;\ \alpha \in \Lambda\}$ be the family of ξ_T from Υ_1 to Υ_2. Then, $\bigcap_{\alpha\in\Lambda} T_\alpha$ is also ξ_T from Υ_1 to Υ_2. Further $\bigcup_{\alpha\in\Lambda} T_\alpha$ need not be ξ_T.

Definition 2.3: Let $(\Upsilon_1, \Upsilon_2, \xi)$ be a $\xi_T S$ and $L \subseteq \Upsilon_1, M \subseteq \Upsilon_2$. Then (L, M) is called ξ-closed in $(\Upsilon_1, \Upsilon_2, \xi)$ if $(\Upsilon_1 \backslash L, \Upsilon_2 \backslash M) \in \xi$.

Proposition 2.1: Let$(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$. Then (Υ_1, Υ_2) and $(\emptyset, \emptyset)$ are ξ-closed sets. Similarly if $\{(L_\alpha, M_\alpha) \colon \alpha \in \Gamma\}$ is a family of ξ-closed sets, then $(\bigcap_{\alpha\in\Gamma} L_\alpha, \bigcap_{\alpha\in\Gamma} M_\alpha)$ is ξ-closed.

Definition 2.4: Let$(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$ and $(L, M) \subseteq (\Upsilon_1, \Upsilon_2)$. Let ${(L, M)^{1^*}}_{\xi} = \bigcap\{L_\alpha \colon (L_\alpha,\ M_\alpha)$ is ξ-closed set and $(L, M) \subseteq (L_\alpha,\ M_\alpha)\}$ and ${(L, M)^{2^*}}_{\xi} = \bigcap\{M_\alpha \colon (L_\alpha,\ M_\alpha)$ is ξ-closed set and $(L, M) \subseteq (L_\alpha,\ M_\alpha)\}$.Then ${(L, M)^{1^*}}_{\xi}, {(L, M)^{2^*}}_{\xi})$ is ξ-closed set and $(L, M) \subseteq (L, M)^{1^*}{}_{\xi}, {(L, M)^{2^*}}_{\xi})$. The ordered pair $\left({(L, M)^{1^*}}_{\xi}, {(L, M)^{2^*}}_{\xi}\right)$ is called *ξ-closure* of (L, M) and is denoted $Cl_\xi(L, M)$ in $\xi_T S$ (X, Y, μ) where $(L, M) \subseteq (\Upsilon_1, \Upsilon_2)$.

Proposition 2.2: Let$(L, M) \subseteq (\Upsilon_1, \Upsilon_2)$. Then (L, M) is ξ-open in $(\Upsilon_1, \Upsilon_2, \xi)$ iff $(L, M) = I_\xi(L, M)$ and (L, M) is ξ-closed in $(\Upsilon_1, \Upsilon_2, \xi)$ iff $(L, M) = Cl_\xi(L, M)$.

Proposition 2.3: Let $(L, M) \subseteq (N, P) \subseteq (\Upsilon_1, \Upsilon_2)$ and $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$. Then $Cl_\xi(\emptyset, \emptyset) = (\emptyset, \emptyset)$, $Cl_\xi(X, Y) = (X, Y)$, $(L, M) \subseteq Cl_\xi(L, M)$, ${(L, M)^{1^*}}_{\xi} \subseteq {(N, P)^{1^*}}_{\xi}$, ${(L, M)^{2^*}}_{\xi}) \subseteq {(N, P)^{2^*}}_{\xi}$, $Cl_\xi(L, M) \subseteq Cl_\xi(N, P)$ and $Cl_\xi(Cl_\xi(L, M)) = Cl_\xi(L, M)$

Definition 2.5: Let$(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$ and $(L, M) \subseteq (\Upsilon_1, \Upsilon_2)$. Let ${(L, M)^{1^0}}_{\xi} = \bigcup\{L_\alpha \colon (L_\alpha,\ M_\alpha)$is ξ-open set and $(L, M) \subseteq (L_\alpha,\ M_\alpha)\}$ and ${(L, M)^{2^0}}_{\xi} = \bigcup\{M_\alpha \colon (L_\alpha,\ M_\alpha)$is ξ-open set and $(L, M) \subseteq (L_\alpha,\ M_\alpha)\}$.Then ${(L, M)^{1^0}}_{\xi}, {(L, M)^{2^0}}_{\xi})$ is ξ-open set and ${(L, M)^{1^0}}_{\xi}, {(L, M)^{2^0}}_{\xi}) \subseteq (L, M)$. The ordered pair $\left({(L, M)^{1^0}}_{\xi}, {(L, M)^{2^0}}_{\xi}\right)$ is called *ξ-interior* of (L, M) and is denoted $I_\xi(L, M)$ in $\xi_T S$ (X, Y, μ) where $(L, M) \subseteq (\Upsilon_1, \Upsilon_2)$.

Proposition 2.4: Let $(L, M) \subseteq (\Upsilon_1, \Upsilon_2)$. Then (L, M) is ξ-open set in $(\Upsilon_1, \Upsilon_2, \xi)$ iff $(L, M) = I_\xi(L, M)$.

Proposition 2.5: Let $(L, M) \subseteq (N, P) \subseteq (\Upsilon_1, \Upsilon_2)$ and $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$. Then $I_\xi(\emptyset, \emptyset) = (\emptyset, \emptyset), I_\xi(X, Y) = (X, Y)$, ${(L, M)^{1^0}}_{\xi} \subseteq {(N, P)^{1^0}}_{\xi}$, ${(L, M)^{2^0}}_{\xi} \subseteq {(N, P)^{2^0}}_{\xi}$, $I_\xi(L, M) \subseteq I_\xi(N, P)$ and $I_\xi(I_\xi(L, M)) = I_\xi(L, M)$

Definition 2.6: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$ and $(Z, \mathcal{T})$ be G_T. Then the map $\mathcal{F} \colon (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is called ξ-continuous at $z \in Z$ if for any ξ-open set $(L, M) \in (\Upsilon_1, \Upsilon_2, \xi)$ with $\mathcal{F}(z) \in (L, M)$ then there exists $\mathcal{T}$-open G in $(Z, \mathcal{T})$ such that $z \in G$ and $\mathcal{F}(G) \subseteq (L, M)$. The mapping $\mathcal{F}$ is called ξ-continuous if it is ξ-continuous at each $z \in Z$.

Proposition 2.6: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$ and $(Z, \mathcal{T})$ be G_T. Then the map $\mathcal{F} \colon (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is called ξ-continuous if $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-open in $(Z, \mathcal{T})$ for every ξ-open set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$.

Definition 2.7: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$ and $(Z, \mathcal{T})$ be G_T. Then the map $\mathcal{F} \colon (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is called *almost perfectly ξ-continuous map $(AP\xi CM)$* if $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-clopen in $(Z, \mathcal{T})$ for every ξ-regular open set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$.

Definition 2.8: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$ and $(Z, \mathcal{T})$ be G_T. Then the map $\mathcal{F} \colon (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is called *completely ξ-continuous map $(C\xi CM)$* if $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-regular open in $(Z, \mathcal{T})$ for every ξ-open set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$.

Definition 2.9: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$ and $(Z, \mathcal{T})$ be G_T. Then the map $\mathcal{F} \colon (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is called *almost completely ξ-continuous map $(AC\xi CM)$* if $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-regular open in $(Z, \mathcal{T})$ for every ξ-regular open set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$.

Definition 2.10: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$ and $(Z, \mathcal{T})$ be G_T. Then the map $\mathcal{F} \colon (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is called *totally ξ-continuous map $(T\xi CM)$* if $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-clopen in $(Z, \mathcal{T})$ for every ξ-open set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$.

Definition 2.11: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$ and $(Z, \mathcal{T})$ be G_T. Then the map $\mathcal{F} \colon (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is called *strongly ξ-continuous map $(S\xi CM)$* if $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-clopen in $(Z, \mathcal{T})$ for every ξ-set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$.

3. Almost Completely ξ-θ-Semi-Continuous Maps (ACξθSCM)

In this section, the relationships of *ξ-θ-semi-continuous maps, almost completely ξ-θ-semi-continuous maps, θ-semi-totally ξ-continuous maps strongly ξ-θ-semi-continuous maps, almost perfectly ξ-θ-semi-continuous maps* and *totally ξ-θ-semi-continuous maps* with each other and some other maps have been established by making the use of some counter examples.

Definition 3.1: Let $(\Upsilon_1, \Upsilon_2, \xi)$ be $\xi_T S$ and $(L, M) \in \wp(\Upsilon_1) \times \wp(\Upsilon_2)$, then

1. $\delta Cl_\xi(L, M) = \{(x, y) \in \wp(\Upsilon_1) \times \wp(\Upsilon_2) : I_\xi\left(Cl_\xi(U, V)\right) \cap (L, M) \neq \emptyset, (U, V) \in \xi \text{ and } (x, y) \in (U, V)\}$
2. $\theta Cl_\xi(L, M) = \{(x, y) \in \wp(\Upsilon_1) \times \wp(\Upsilon_2) : Cl_\xi(U, V) \cap (L, M) \neq \emptyset, (U, V) \in \xi \text{ and } (x, y) \in (U, V)\}$

Definition 3.2: Let $(\Upsilon_1, \Upsilon_2, \xi)$ be $\xi_T S$. Then set (L, M) is called ξ-θ-semi-open set $(L, M) \subseteq Cl_\xi(\theta I_\xi(L, M))$

Definition 3.2: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$ and $(Z, \mathcal{T})$ be G_T. Then the map $\mathcal{F} : (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is called *ξ-θ-semi-continuous map* (*$\xi\theta SCM$*) if $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-θ-semi-open in $(Z, \mathcal{T})$ for every ξ-open set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$.

Definition 3.3: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$ and $(Z, \mathcal{T})$ be G_T. Then the map $\mathcal{F} : (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is called *almost completely ξ-θ-semi-continuous map* (*$AC\xi\theta SCM$*) if $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-θ-semi-open in $(Z, \mathcal{T})$ for every ξ-θ-semi-open set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$.

Remark 3.1: $\xi\theta SCM \not\Leftrightarrow AC\xi\theta SCM$

The result can be illustrated in Example 3.1 and Example 3.2.

Example 3.1: Let $Z = \{1,2,3\}$, $\Upsilon_1 = \{m_1, m_2\}$ and $\Upsilon_2 = \{l_1, l_2\}$. Then $\mathcal{T} = \{\emptyset, \{1\}, \{1,2\}, \{2,3\}, Z\}$ and $\xi = \{(\emptyset, \emptyset), (\{m_1\}, \{l_1\}), (\{m_2\}, \{l_2\}), (\{m_2\}, \{\Upsilon_2\}), (\Upsilon_1, \Upsilon_2)\}$. Clearly $\mathcal{T}$ is G_T on Z and ξ is ξ_T from Υ_1 to Υ_2. Define $\mathcal{F} : (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ by $\mathcal{F}(1) = (m_1, l_2)$, $\mathcal{F}(2) = (m_1, l_1)$ and $\mathcal{F}(3) = (m_2, l_2)$. This shows that the inverse image of every ξ-open set in $(\Upsilon_1, \Upsilon_2, \xi)$ is $\mathcal{T}$-θ-semi-open in $(Z, \mathcal{T})$. Hence $\mathcal{F} : (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is ξθSCM but not ACξθSCM, because $\mathcal{F}^{-1}(\{m_1\}, \{l_2\}) = \{1\}$, where $\{1\}$ is not $\mathcal{T}$- θ-semi-open set in $(Z, \mathcal{T})$.

Example 3.2: Let $Z = \{1,2,3\}$, $\Upsilon_1 = \{m_1, m_2\}$ and $\Upsilon_2 = \{l_1, l_2\}$. Then $\mathcal{T} = \{\emptyset, \{2\}, \{1,3\}, \{2,3\}, Z\}$ and $\xi = \{(\emptyset, \emptyset), (\{m_1\}, \{l_1\}), (\{m_2\}, \{l_2\}), (\{m_2\}, \{\Upsilon_2\}), (\Upsilon_1, \Upsilon_2)\}$. Clearly $\mathcal{T}$ is G_T on Z and ξ is ξ_T from Υ_1 to Υ_2. Define $\mathcal{F} : (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ by $\mathcal{F}(1) = (m_1, l_1)$, $\mathcal{F}(2) = (m_1, l_2)$ and $\mathcal{F}(3) = (m_2, l_2)$. This shows that the inverse image of every ξ-θ-semi-open set in $(\Upsilon_1, \Upsilon_2, \xi)$ is $\mathcal{T}$-θ-semi-open in $(Z, \mathcal{T})$. Hence $\mathcal{F} : (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is ACξθSCM but not ξθSCM, because $\mathcal{F}^{-1}(\{m_1\}, \{l_1\}) = \{1\}$, where $\{1\}$ is not $\mathcal{T}$- θ-semi-open set in $(Z, \mathcal{T})$.

Definition 3.4: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$ and $(Z, \mathcal{T})$ be G_T. Then the map $\mathcal{F} : (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is called *θ-semi-totally ξ-continuous map* (*$\theta ST\xi CM$*) if $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-clopen in $(Z, \mathcal{T})$ for every ξ-θ-semi-open set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$.

Remark 3.2: $\theta ST\xi CM \not\Leftrightarrow AC\xi\theta SCM$

The result can be illustrated in Example 3.3 and Example 3.4.

Example 3.3: Let $Z = \{1,2,3\}$, $\Upsilon_1 = \{m_1, m_2\}$ and $\Upsilon_2 = \{l_1, l_2\}$. Then $\mathcal{T} = \{\emptyset, \{1\}, \{1,2\}, \{2,3\}, Z\}$ and $\xi = \{(\emptyset, \emptyset), (\{m_1\}, \{l_1\}), (\{m_2\}, \{l_2\}), (\{m_2\}, \{\Upsilon_2\}), (\Upsilon_1, \Upsilon_2)\}$. Clearly $\mathcal{T}$ is G_T on Z and ξ is ξ_T from Υ_1 to Υ_2. Define $\mathcal{F} : (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ by $\mathcal{F}(1) = (m_1, l_2)$ and $\mathcal{F}(2) = (m_1, l_1) = \mathcal{F}(3)$. This shows that the inverse image of every ξ-θ-semi-open set in $(\Upsilon_1, \Upsilon_2, \xi)$ is $\mathcal{T}$-clopen in $(Z, \mathcal{T})$. Hence $\mathcal{F} : (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is θSTξCM but not ACξθSCM, because $\mathcal{F}^{-1}(\{m_1\}, \{l_2\}) = \{1\}$, where $\{1\}$ is $\mathcal{T}$-clopen but not $\mathcal{T}$- θ-semi-open set in $(Z, \mathcal{T})$.

Example 4.4: Let $Z = \{1,2,3\}$, $\Upsilon_1 = \{m_1, m_2\}$ and $\Upsilon_2 = \{l_1, l_2\}$. Then $\mathcal{T} = \{\emptyset, \{2\}, \{1,3\}, \{2,3\}, Z\}$ and $\xi = \{(\emptyset, \emptyset), (\{m_1\}, \{l_1\}), (\{m_2\}, \{l_2\}), (\{m_2\}, \{\Upsilon_2\}), (\Upsilon_1, \Upsilon_2)\}$. Clearly $\mathcal{T}$ is G_T on Z and ξ is ξ_T from Υ_1 to Υ_2. Define $\mathcal{F} : (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ by $\mathcal{F}(1) = (m_1, l_1)$, $\mathcal{F}(2) = (m_2, l_2)$ and $\mathcal{F}(3) = (m_1, l_2)$. This shows that the inverse image of every ξ-θ-semi-open set in $(\Upsilon_1, \Upsilon_2, \xi)$ is $\mathcal{T}$- θ-semi-open set in $(Z, \mathcal{T})$. Hence $\mathcal{F} : (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is ACξθSCM but not θSTξCM, because $\mathcal{F}^{-1}(\{m_1\}, \{l_2\}) = \{3\}$, where $\{3\}$ is $\mathcal{T}$- θ-semi-open set but not $\mathcal{T}$-clopen in $(Z, \mathcal{T})$.

Definition 3.5: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$ and $(Z, \mathcal{T})$ be G_T. Then the map $\mathcal{F} : (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is called *strongly ξ-θ-semi-continuous map* (*$S\xi\theta SCM$*) if $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$- θ-semi-clopen in $(Z, \mathcal{T})$ for every ξ-set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$.

Remark 3.3: $S\xi\theta SCM \Rightarrow\not\Leftarrow AC\xi\theta SCM$

Proof: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$ and $(Z, \mathcal{T})$ be G_T and the map $\mathcal{F} : (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is SξθSCM. Let (L, M) be ξ- θ-semi-open set in $(\Upsilon_1, \Upsilon_2, \xi)$. Since $\mathcal{F} : (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is SξθSCM, therefore $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$- θ-semi-clopen or $\mathcal{T}$-θ-semi-open in $(Z, \mathcal{T})$ for every ξ- θ-semi-open set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$. Hence $\mathcal{F} : (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is ACξθSCM.

The converse can be illustrated in Example 3.5.

Example 3.5: Let $Z = \{1,2,3\}$, $\Upsilon_1 = \{m_1, m_2\}$ and $\Upsilon_2 = \{l_1, l_2\}$. Then $\mathcal{T} = \{\emptyset, \{1,2\}, \{2,3\}, Z\}$ and $\xi = \{(\emptyset, \emptyset), (\{m_1\}, \{l_1\}), (\{m_2\}, \{l_2\}), (\{m_2\}, \{\Upsilon_2\}), (\Upsilon_1, \Upsilon_2)\}$. Clearly $\mathcal{T}$ is G_T on Z and ξ is ξ_T from Υ_1 to Υ_2. Define $\mathcal{F} : (Z, \mathcal{T}) \to$

$\Upsilon_1 \times \Upsilon_2$ by $\mathcal{F}(1) = (m_1, l_2)$, $\mathcal{F}(2) = (m_1, l_1)$ and $\mathcal{F}(3) = (m_2, \emptyset)$. This shows that the inverse image of every ξ-θ-semi-open set in $(\Upsilon_1, \Upsilon_2, \xi)$ is $\mathcal{T}$- θ-semi-open set in $(Z, \mathcal{T})$. Hence $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is ACξθSCM but not SξθSCM, because $\mathcal{F}^{-1}(\{m_1\}, \{l_2\}) = \{1\}$, where $\{3\}$ is $\mathcal{T}$- θ-semi-open set but not $\mathcal{T}$- θ-semi-clopen in $(Z, \mathcal{T})$.

Definition 3.6: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$ and $(Z, \mathcal{T})$ be G_T. Then the map $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is called *almost perfectly ξ-θ-semi-continuous map* $(AP\xi\theta SCM)$ if $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-clopen in $(Z, \mathcal{T})$ for every ξ-θ-semi-open set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$.

Remark 3.4: ξθSCM ⇎ APξθSCM

The result can be illustrated in Example 3.6 and Example 3.7.

Example 3.6: Let $Z = \{1,2,3\}$, $\Upsilon_1 = \{m_1, m_2\}$ and $\Upsilon_2 = \{l_1, l_2\}$. Then $\mathcal{T} = \{\emptyset, \{1\}, \{1,2\}, \{2,3\}, Z\}$ and $\xi = \{(\emptyset, \emptyset), (\{m_1\}, \{l_1\}), (\{m_2\}, \{l_2\}), (\{m_2\}, \{\Upsilon_2\}), (\Upsilon_1, \Upsilon_2)\}$. Clearly $\mathcal{T}$ is G_T on Z and ξ is ξ_T from Υ_1 to Υ_2. Define $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ by $\mathcal{F}(1) = (m_1, l_2) = \mathcal{F}(3)$ and $\mathcal{F}(2) = (m_1, l_1)$. Thus that the inverse image of every ξ-θ-semi-open set in $(\Upsilon_1, \Upsilon_2, \xi)$ is $\mathcal{T}$- θ-semi-open set in $(Z, \mathcal{T})$. Hence $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is ξθSCM but not APξθSCM, because $\mathcal{F}^{-1}(\{m_1\}, \{l_2\}) = \{1,3\}$, where $\{1,3\}$ is $\mathcal{T}$- θ-semi-open set but not $\mathcal{T}$-clopen in $(Z, \mathcal{T})$.

Example 3.7: Let $Z = \{1,2,3\}$, $\Upsilon_1 = \{m_1, m_2\}$ and $\Upsilon_2 = \{l_1, l_2\}$. Then $\mathcal{T} = \{\emptyset, \{2\}, \{1,3\}, \{2,3\}, Z\}$ and $\xi = \{(\emptyset, \emptyset), (\{m_1\}, \{l_1\}), (\{m_2\}, \{l_2\}), (\{m_2\}, \{\Upsilon_2\}), (\Upsilon_1, \Upsilon_2)\}$. Clearly $\mathcal{T}$ is G_T on Z and ξ is ξ_T from Υ_1 to Υ_2. Define $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ by $\mathcal{F}(1) = (m_1, l_1) = \mathcal{F}(3)$ and $\mathcal{F}(2) = (m_1, \emptyset)$. Thus that the inverse image of every ξ-θ-semi-open set in $(\Upsilon_1, \Upsilon_2, \xi)$ is $\mathcal{T}$-clopen set in $(Z, \mathcal{T})$. Hence $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is APξθSCM but not ξθSCM, because $\mathcal{F}^{-1}(\{m_1\}, \{l_2\}) = \{1,3\}$, where $\{1,3\}$ is $\mathcal{T}$-clopen but not $\mathcal{T}$- θ-semi-open set in $(Z, \mathcal{T})$.

Remark 3.5: SξθSCM ⇎ APξθSCM

The result can be illustrated in Example 3.8 and Example 4.9.

Example 3.8: Let $Z = \{1,2,3\}$, $\Upsilon_1 = \{m_1, m_2\}$ and $\Upsilon_2 = \{l_1, l_2\}$. Then $\mathcal{T} = \{\emptyset, \{1,2\}, \{2,3\}, Z\}$ and $\xi = \{(\emptyset, \emptyset), (\{m_1\}, \{l_1\}), (\{m_2\}, \{l_2\}), (\{m_2\}, \{\Upsilon_2\}), (\Upsilon_1, \Upsilon_2)\}$. Clearly $\mathcal{T}$ is G_T on Z and ξ is ξ_T from Υ_1 to Υ_2. Define $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ by $\mathcal{F}(1) = (m_1, l_2) = \mathcal{F}(3)$ and $\mathcal{F}(2) = (m_1, l_1)$. Thus that the inverse image of every ξ-set in $(\Upsilon_1, \Upsilon_2, \xi)$ is $\mathcal{T}$- θ-semi-open set in $(Z, \mathcal{T})$. Hence $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is SξθSCM but not APξθSCM, because $\{1,3\}$and $\{2\}$ are $\mathcal{T}$- θ-semi-clopen but not $\mathcal{T}$-clopen in $(Z, \mathcal{T})$.

Example 3.9: Let $Z = \{1,2,3\}$, $\Upsilon_1 = \{m_1, m_2\}$ and $\Upsilon_2 = \{l_1, l_2\}$. Then $\mathcal{T} = \{\emptyset, \{2\}, \{1,3\}, \{2,3\}, Z\}$ and $\xi = \{(\emptyset, \emptyset), (\{m_1\}, \{l_1\}), (\{m_2\}, \{l_2\}), (\{m_2\}, \{\Upsilon_2\}), (\Upsilon_1, \Upsilon_2)\}$. Clearly $\mathcal{T}$ is G_T on Z and ξ is ξ_T from Υ_1 to Υ_2. Define $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ by $\mathcal{F}(1) = (m_1, l_2) = \mathcal{F}(3)$ and $\mathcal{F}(2) = (m_1, \emptyset)$. Thus that the inverse image of every ξ-θ-semi-open set in $(\Upsilon_1, \Upsilon_2, \xi)$ is $\mathcal{T}$-clopen set in $(Z, \mathcal{T})$. Hence $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is APξθSCM but not SξθSCM, because $\mathcal{F}^{-1}(\{m_1\}, \{l_2\}) = \{1,3\}$, where $\{1,3\}$ is $\mathcal{T}$-clopen but not $\mathcal{T}$- θ-semi-open set in $(Z, \mathcal{T})$.

Definition 3.7: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$ and $(Z, \mathcal{T})$ be G_T. Then the map $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is called *totally ξ-θ-semi-continuous map* $(T\xi\theta SCM)$ if $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$- θ-semi-clopen in $(Z, \mathcal{T})$ for every ξ-open set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$.

Remark 3.6: TξθSCM ⇎ ACξθSCM

The result can be illustrated in Example 3.10 and Example 3.11.

Example 3.10: Let $Z = \{1,2,3\}$, $\Upsilon_1 = \{m_1, m_2\}$ and $\Upsilon_2 = \{l_1, l_2\}$. Then $\mathcal{T} = \{\emptyset, \{1\}, \{1,2\}, \{2,3\}, Z\}$ and $\xi = \{(\emptyset, \emptyset), (\{m_1\}, \{l_1\}), (\{m_2\}, \{l_2\}), (\{m_2\}, \{\Upsilon_2\}), (\Upsilon_1, \Upsilon_2)\}$. Clearly $\mathcal{T}$ is G_T on Z and ξ is ξ_T from Υ_1 to Υ_2. Define $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ by $\mathcal{F}(1) = (m_1, l_2)$, $\mathcal{F}(2) = (m_1, l_1)$ and $\mathcal{F}(3) = (m_1, \emptyset)$. Thus that the inverse image of every ξ-open set in $(\Upsilon_1, \Upsilon_2, \xi)$ is $\mathcal{T}$- θ-semi-clopen set in $(Z, \mathcal{T})$. Hence $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is TξθSCM but not ACξθSCM, because $\mathcal{F}^{-1}(\{m_1\}, \{l_2\}) = \{1\}$, where $\{1\}$ is not $\mathcal{T}$- θ-semi-open set in $(Z, \mathcal{T})$.

Example 3.11: Let $Z = \{1,2,3\}$, $\Upsilon_1 = \{m_1, m_2\}$ and $\Upsilon_2 = \{l_1, l_2\}$. Then $\mathcal{T} = \{\emptyset, \{1,2\}, \{2,3\}, Z\}$ and $\xi = \{(\emptyset, \emptyset), (\{m_1\}, \{l_1\}), (\{m_2\}, \{l_2\}), (\{m_2\}, \{\Upsilon_2\}), (\Upsilon_1, \Upsilon_2)\}$. Clearly $\mathcal{T}$ is G_T on Z and ξ is ξ_T from Υ_1 to Υ_2. Define $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ by $\mathcal{F}(1) = (m_1, l_2)$, $\mathcal{F}(2) = (m_1, \emptyset)$ and $\mathcal{F}(3) = (\emptyset, l_1)$. Thus that the inverse image of every ξ-θ-semi-open set in $(\Upsilon_1, \Upsilon_2, \xi)$ is $\mathcal{T}$- θ-semi-open set in $(Z, \mathcal{T})$. Hence $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is ACξθSCM but not TξθSCM, because $\mathcal{F}^{-1}(\{m_1\}, \{l_2\}) = \{1\}$, where $\{1\}$ is $\mathcal{T}$- θ-semi-open set but not $\mathcal{T}$- θ-semi-clopen set in $(Z, \mathcal{T})$.

Remark 3.7: TξθSCM ⇎ APξθSCM

The result can be illustrated in Example 3.12 and Example 3.13.

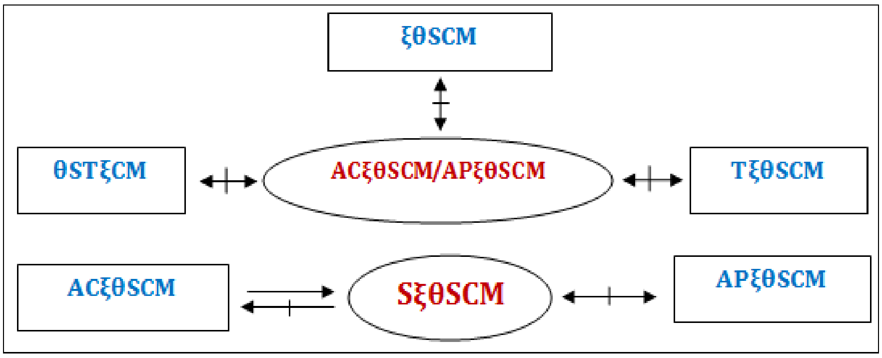
Example 3.12: Let $Z = \{1,2,3\}$, $\Upsilon_1 = \{m_1, m_2\}$ and $\Upsilon_2 = \{l_1, l_2\}$. Then $\mathcal{T} = \{\emptyset, \{1,2\}, \{2,3\}, Z\}$ and $\xi = \{(\emptyset, \emptyset), (\{m_1\}, \{l_1\}), (\{m_2\}, \{l_2\}), (\{m_2\}, \{\Upsilon_2\}), (\Upsilon_1, \Upsilon_2)\}$. Clearly $\mathcal{T}$ is G_T on Z and ξ is ξ_T from Υ_1 to Υ_2. Define $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ by $\mathcal{F}(1) = (m_1, l_1)$, $\mathcal{F}(2) = (m_1, \emptyset)$ and $\mathcal{F}(3) = (m_1, l_2)$. Thus that the inverse image of every ξ-open set in $(\Upsilon_1, \Upsilon_2, \xi)$ is $\mathcal{T}$- θ-semi-clopen set in $(Z, \mathcal{T})$. Hence $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ isTξθSCM but not APξθSCM, because $\mathcal{F}^{-1}(\{m_1\}, \{l_2\}) = \{3\}$, where $\{3\}$ is not $\mathcal{T}$-clopen set in $(Z, \mathcal{T})$.

Example 3.13: Let $Z = \{1,2,3\}$, $\Upsilon_1 = \{m_1, m_2\}$ and $\Upsilon_2 = \{l_1, l_2\}$. Then $\mathcal{T} = \{\emptyset, \{2\}, \{1,3\}, \{2,3\}, Z\}$ and $\xi =$

$\{(\emptyset,\emptyset),(\{m_1\},\{l_1\}),(\{m_2\},\{l_2\}),(\{m_2\},\{\Upsilon_2\}),(\Upsilon_1,\Upsilon_2)\}$. Clearly $\mathcal{T}$ is G_T on Z and ξ is ξ_T from Υ_1 to Υ_2. Define $\mathcal{F}:(Z,\mathcal{T})\to\Upsilon_1\times\Upsilon_2$ by $\mathcal{F}(1)=(m_1,l_1)=\mathcal{F}(3)$ and $\mathcal{F}(2)=(m_1,l_2)$. Thus that the inverse image of every ξ- θ-semi-open set in $(\Upsilon_1,\Upsilon_2,\xi)$ is $\mathcal{T}$-clopen set in $(Z,\mathcal{T})$. Hence $\mathcal{F}:(Z,\mathcal{T})\to\Upsilon_1\times\Upsilon_2$ is APξθSCM but not TξθSCM, because $\mathcal{F}^{-1}(\{m_1\},\{l_2\})=\{1,3\}$, where $\{1,3\}$ is $\mathcal{T}$-clopen set but not $\mathcal{T}$- θ-semi-clopen set in $(Z,\mathcal{T})$.

6. CONCLUSION

In this paper, a very useful concept of ξ-θ-semi-continuous maps, almost completely ξ-θ-semi-continuous maps, θ-semi-totally ξ-continuous maps strongly ξ-θ-semi-continuous maps, almost perfectly ξ-θ-semi-continuous maps, totally ξ-θ-semi-continuous maps have been introduced and established the relationships between these maps with some other maps. In the present direction, we have categorised maps in generalized binary topological spaces and investigated the behaviour of presented maps with the help of appropriate examples. The conclusion is illustrated by the following figure



References

1. Arya,S. P. and Gupta,R. On strongly continuous functions, Kyungpook Math. J., 14, 131-143, (1974)
2. Anuradha N. and Baby Chacko, Some Properties of Almost Perfectly Continuous Functions in Topological Spaces, International Mathematical Forum 10(3), 143-156 (2015)
3. Benchalli **S.S.** and Umadevi I Neeli “Semi-Totally Continuous Functions in Topological Spaces” International Mathematical Forum 6(10), 479-492, (2011)
4. Bhattacharya,S, On Generalized Regular Closed Sets ,Int . J. Contemp. Math. Sciences, 6 (3) 145-152 (2011).
5. Chen, C.C., Conejero, J.A., Kostic, M., Murillo-Arcila., M., Dynamics on Binary Relations over Topological Spaces. Symmetry 2018, 10: 211. https://doi.org/10.3390/sym10060211
6. Csaszar, A. Generalized topology, generalized continuity, Acta Math. Hungar, 96, 351-357 (2002)
7. Egenhofer, MJ. Reasoning about binary topological relations. Symposium on Spatial Databases SSD 1991: Advances in Spatial Databases, 141-160, (1991)
8. Engelking R. Generel Topology, Polish Scientific Publishers, Warszawa (1977)
9. Gevorgyan, PS. Groups of binary operations and binary G-spaces. Topology and its Applications, 201, 18–28, (2016)
10. Hatir E, Noiri T. Decompositions of continuity and complete continuity. Acta Math Hungary , 4, 281–287, (2006)
11. Jamal M. Mustafa, On Binary Generalized Topological Spaces, Refaad General Letters in Mathematics, 2(3), 111-116 (2017)
12. Kırbas, H., Aslım, G. Decompositions of continuity and some weak forms of continuity. Chaos, Solitons and Fractals, 41, 1684–1690, (2009).
13. Khalimsky, ED, Kopperman R, Meyer PR. Computer graphics and connected topologies an finite ordered sets. TopolAppl, 36, 1–17, (1990).
14. Levine N. A decomposition of continuity in topological spaces. Am Math Mon, 68, 44–6, (1961)
15. Levine, N. Semi open sets and semi continuity in topological spaces, Amer. Math. Monthly, 70, 36-41, (1963).
16. Levine, N. Generalized closed sets in Topology, Rend. Cir. Mat. Palermo, 2, 89-96, (1970).

17. Njastad, O, On some classes of nearly open sets, Pacific J. Math, 15, 961–970, (1965).
18. NithyananthaJothi S., and P. Thangavelu, Topology between two sets, Journal of Mathematical Sciences & Computer Applications, 1(3), 95-107 (2011)
19. Nour T.M, Totally semi-continuous functions, Indian J. Pure Appl.Math, 26(7), 675 – 678 (1995)
20. Pawlak, Z. Rough sets: theoretical aspects of reasoning about data. System theory, knowledge engineering and problem solving, vol. 9. Dordrecht: Kluwer; (1991).
21. Singh D., Almost Perfectly continuous functions, Quaest Math , 33, 1-11 (2010)
22. Son MJ, Park JH, Lim KM. Weakly clopen functions. Chaos, Solitons& Fractals, 33, 1746–55, (2007)
23. Svozil, K. Quantum field theory on fractal space–time: a new regularization method. J Phys A Math Gen, 20, 3861–75, (1987).